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On the incremental torsional stiffness of an annular disc bonded to a finitely deformed elastic halfspace

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Abstract

This paper investigates the elastostatic problem of an annular rigid disc bonded to a finitely deformed incompressible elastic halfspace. The analysis is developed within the context of the theory of small deformations superposed on large proposed by Green, Rivlin and Shield (1952). The triple integral equations encountered in the formulation of the mixed boundary value problems are solved in an approximate fashion in terms of series involving a small non-dimensional parameter that corresponds to the radii ratio for the annulus. Explicit results are provided for the torsional stiffness of the annular rigid indenter in terms of the homogeneous finite deformation and the constitutive properties of the incompressible rubberlike elastic solid.

Keywords: superposed small deformations, annular indenter, incompressible elastic solid, rubber-like materials, triple integral equations, integral transform methods

1. Introduction

Theories that describe the superposition of small deformations on an initial finite deformation of an elastic body have been proposed by a number of researchers including Trefftz (1933), Biot (1939), Neuber (1943) and Green et al. (1952). The theory proposed by Green et al. (1952) is recognized for its rigorous basis for accommodating the initial finite deformation within the context of modern theories of rubber elasticity (Rivlin, 1960; Spencer, 1970; Ogden, 1984; Rajagopal, 1995; Barenblatt and Joseph, 1997; Selvadurai, 2006). Detailed expositions of the general theory of small deformations superposed on large can also be found in the texts by Truesdell and Noll (1965), Green and Zerna (1968), Green and Adkins (1970) and Eringen and Suhubi (1975).

This paper uses the theory of small deformations superposed on finite deformations developed by Green et al. (1952) to examine the axisymmetric problem arising from the torsional indentation of the surface of an initially finitely deformed incompressible elastic halfspace by an annular rigid indenter bonded to the surface. The theory of small deformations superposed on large is a useful model for treating hyperelastic materials that are pre-stressed as opposed to continuously undergoing moderately large deformations similar to the theory of second-order elasticity (see e.g. Rivlin, 1953; Green and Spratt, 1954; Selvadurai and Spencer, 1972; Selvadurai et al., 1988). The analysis of the three-part mixed boundary value problems associated with the indentation problems can be formulated by appeal to Hankel transform development of the governing equations. This particular approach has been successfully applied in the literature to examine a large class of boundary value problems associated with fundamental solutions (Woo and Shields, 1961), flexible contact problems (Selvadurai, 1977), rigid indenters (Beatty and Usmani, 1975), rubber mounting problems (Hill, 1975 a, b; 1976, 1977) and crack problems (Selvadurai, 1980). Also Demiray (1992) and more recently Baek et al.(2007) have applied the theory of small deformation superposed on large to examine biomechanics problems related to arteries. In this paper we examine the problem of a incompressible rubber-like elastic half space which is first subjected to a finite radial strain and subsequently subjected to a state of axisymmetric torsion by an annular disc that is

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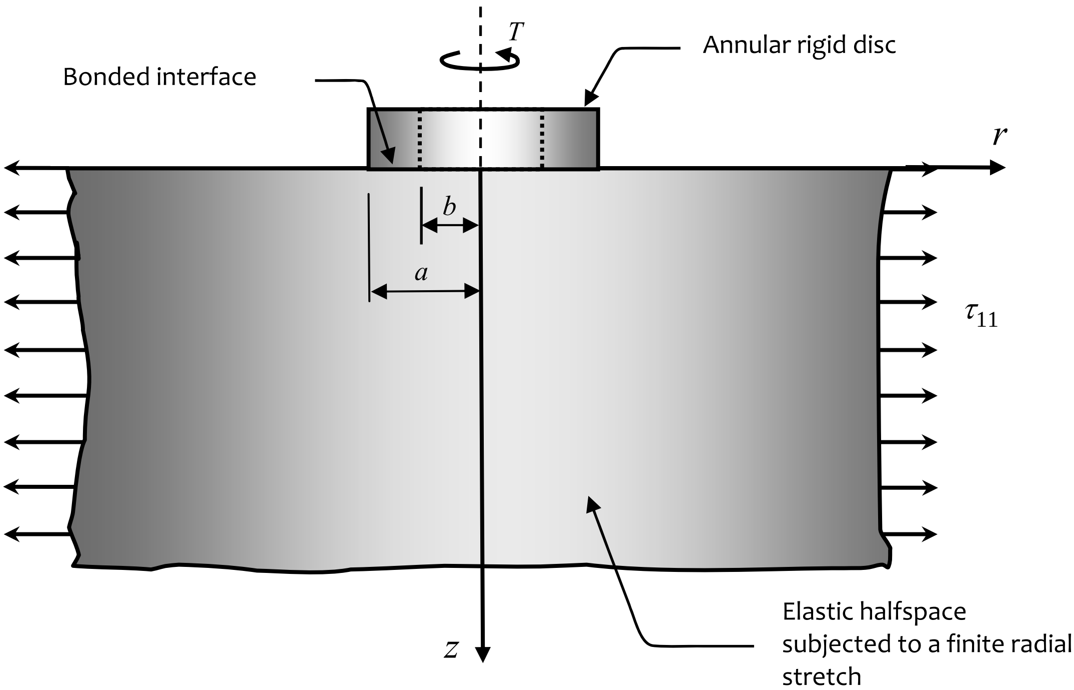

Figure 1. Torsional indentation of a finitely stretched incompressible elastic halfspace by a bonded annular rigid disc

bonded to the surface (Figure 1). The objective of the study is to examine the influence of both the finite stretch and the constitutive properties of the hyperelastic material on the torsional stiffness of the bonded annular disc. The study can be used as an experimental procedure for the parameter identification of the constitutive properties governing finite deformations of the hyperelastic material.

2. Governing Equations

The fundamental equations governing small elastic deformations of an incompressible isotropic elastic material subjected to an initial finite deformation are given by Green et al. (1952) and the salient results are summarized for completeness. We define material points in an isotropic elastic medium by a general curvilinear coordinate system $\theta_i \; (\theta_1 = r\, ; \theta_2 = \theta\, ; \theta_3 = z\,)$, which moves with the body as it deforms. The covariant and contravariant metric tensors associated with the undeformed and deformed states are defined by g_{ij} , G_{ij} and g^{ij} , G^{ij} , respectively. For an isotropic, incompressible elastic material, we can define a contravariant stress tensor τ^{ij} , measured per unit area of the deformed body but referred to θ_i coordinates of the deformed body, and the hyperelastic constitutive relationship governing the material can be written as

$$\tau^{ij} = \Phi g^{ij} + \Psi B^{ij} + pG^{ij} \tag{1}$$

where p is an isotropic stress to be determined by satisfying the boundary conditions and

$$B^{ij} = I_1 g^{ij} - g^{ir} g^{js} G_{rs} \tag{2}$$

$$\Phi = 2\frac{\partial W}{\partial I_1} \quad ; \quad \Psi = 2\frac{\partial W}{\partial I_2} \tag{3}$$

In (3), $W = W(I_1, I_2)$ is the strain energy per unit defined per unit volume and $I_n (n = 1,2)$ are the invariants given by

$$I_1 = g^{rs} G_{rs} \quad ; \quad I_2 = g_{rs} G^{rs} \quad ; \quad I_3 = \left| G_{ij} \right| \Big/ \left| g_{ij} \right| = 1 \tag{4}$$

We consider the class of finite deformations, where a halfspace region is subjected to a finite radial stretch μ, maintaining the surface of the halfspace, $z = 0$, traction free. For this finite deformation and the specified traction boundary condition, the stress components τ^{ij} are given by

$$\tau^{11} = r^2 \tau^{22} = \left\{ \mu^2 - \frac{1}{\mu^4} \right\} (\Phi + \mu^2 \Psi) \tag{5}$$

For the solution of the title problem, we consider a superposed state of infinitesimal deformation which is axially symmetric and defined by

$$u_r(r,\theta,z) = 0 \quad ; \quad u_\theta(r,\theta,z) = v(r,z) \quad ; \quad u_z(r,\theta,z) = 0 \tag{6}$$

We note that if the infinitesimal displacement vector only has an azimuthal component and the state of deformation is axisymmetric, the displacement field has to correspond to (6) and any non-zero contributions to u_r and u_z must necessarily be higher-order effects that can be neglected in the context of the superposed small deformations. The non-zero components of the incremental stress tensor $\tilde{\tau}^{ij}(r,z)$ can be obtained in the forms

$$\tilde{\tau}^{11} = r^2 \tilde{\tau}^{22} = \tilde{\tau}^{33} = \tilde{p} \tag{7}$$

$$r\tilde{\tau}^{12} = \alpha_6 \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) \quad ; \quad r\tilde{\tau}^{23} = \alpha_7 \left(\frac{\partial v}{\partial z} \right) \tag{8}$$

where $\tilde{p}$ is an incremental isotropic stress and

$$\alpha_6 = \left(\frac{1}{\mu^4} (\Phi + 2\mu^2 \Psi) - \mu^4 \Psi \right) \quad ; \quad \alpha_7 = \frac{1}{\mu^4} (\Phi + \mu^2 \Psi) \tag{9}$$

In (7) and (9) we have retained, for consistency, the notations for the elastic constants introduced by Green et al. (1952). There are no restrictions imposed on the constants α_6 and α_7 except for the positive definiteness of the strain energy function and the material constraint of incompressibility. The formulation is valid for all classes of hyperelastic materials where the strain energy function depends only on the principal invariants I_1 and I_2.

For rotationally symmetric problems where the small deformations are superposed on an initial homogeneous finite deformation induced by only a radial stress, and in the absence of body forces , the equations of equilibrium governing the superposed stress state $\tilde{\tau}^{ij}$ can be written as

$$\frac{\partial \tilde{p}}{\partial r} = \frac{\partial \tilde{p}}{\partial z} = 0 \tag{10}$$

$$\frac{\partial \tilde{\tau}^{12}}{\partial r}+\frac{\partial \tilde{\tau}^{23}}{\partial z}+\frac{3\tilde{\tau}^{12}}{r}=\frac{\tau^{11}}{r}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right) \tag{11}$$

The formulation of the problem governing the small deformations superposed on large can be completed by specifying appropriate displacement and traction boundary conditions. the specification of the displacement boundary conditions is straightforward and the traction boundary conditions corresponding to the superposed stress field are given by

$$\tilde{\tau}^{ij} n_i + \tau^{ij}\tilde{n}_i = \tilde{P}^j \tag{12}$$

where n_i are the covariant components of the unit normal referred to a surface in the finitely deformed body; $\tilde{n}_i$ and $\tilde{P}^j$ are, respectively, the covariant component of the unit normal and the contravariant component of the surface force vector referred to a boundary in the finitely deformed body.

3. Rotationally Symmetric Incremental Deformations

The equilibrium equations (10) ensure that the incremental isotropic stress can be set to zero without loss of generality. For the solution of the non-trivial equation of equilibrium (11), we can employ the Hankel transform technique outlined by Sneddon (1944, 1951). The first-order Hankel transform of the function $\varphi(r)$ is defined by

$$\bar{\varphi}^1(\xi) = H_1\{\varphi(r);\xi\} \tag{13}$$

where

$$H_1\{\varphi(r);\xi\} = \int_0^\infty r\varphi(r)\, J_1(\xi r)\, dr \tag{14}$$

and the appropriate Hankel inversion theorem is

$$\varphi(r) = \int_0^\infty \xi\, \bar{\varphi}^{(1)}(\xi)\, J_1(\xi r)\, d\xi \tag{15}$$

Operating on (11) with the first-order Hankel transform, we obtain second-order ordinary differential equation for $\bar{v}^1(\xi)$, the solution of which, applicable to a half-space region $0 \le z < \infty$, is given by

$$\bar{v}^1(\xi, z) = A(\xi)\, e^{-(\xi z/k)} \tag{16}$$

where $A(\xi)$ is an arbitrary function and

$$k = \left(\frac{\alpha_7}{\alpha_6+\tau^{11}}\right)^{1/2} = \left(\frac{\Phi+\mu^2\Psi}{\mu^2(\mu^2\Phi+\Psi)}\right)^{1/2} \tag{17}$$

As is evident, the application of the radial stretch induces an apparent structural change in the mechanical behaviour of the material, in that the originally isotropic material now displays traits of transversely isotropic behaviour. When $\mu = 1$, the finite deformation vanishes and the formulation reduces to that of the classical elasticity problem that exhibits azimuthal deformations with axial symmetry.

4. The Torsion of an Annular Rigid Disc Bonded to the Finitely Deformed Halfspace

The problem of the torsional oscillations of a rigid circular disc bonded to the surface of a halfspace was first examined by Reissner and Sagoci (1944) using an oblate spheroidal coordinate formulation, which provides the solution to the rigid torsional oscillations of the rigid disc as a special case. The problem was re-formulated by Sneddon (1944) who applied the theory of dual integral equations to solve the analogous problem for the rigid disc bonded to the surface of a halfspace. The static Reissner-Sagoci problem has been investigated by a number of investigators to include finite dimensions of the elastic medium, transverse isotropy of the elastic halfspace and elastic non-homogeneity of the shear modulus, which is the single material parameter that influences the torsional response. Detailed accounts of these developments can be found in volume by Gladwell (1980) and the articles by Selvadurai (1982, 2007), Selvadurai et al. (1986) and Gladwell and Lemczyk (1990). This paper extends the statical Reissner-Sagoci problem to include the effects of an axisymmetric finite radial stretch of an incompressible elastic halfspace. It should be noted that the surface of the halfspace is initially unconstrained during the application of the finite deformation and the annular rigid disc, which is subsequently bonded to the surface, is subjected to the incremental rotation.

We consider the problem of an annular rigid circular disc of external radius a and internal radius b that is bonded to the surface of an incompressible elastic halfspace that is subjected to a finite stretch μ. The annular rigid disc is subjected to an axisymmetric rotation Ω. The mixed boundary conditions governing the contact problem are

$$\overline{\tau}^{23}(r,0)=0 \quad ; \quad 0<r<b \tag{18}$$

$$v(r,0)=\Omega r \quad ; \quad b\le r\le a \tag{19}$$

$$\overline{\tau}^{23}(r,0)=0 \quad ; \quad b<r<\infty \tag{20}$$

The mixed boundary conditions (18) to (20) gives the following set of triple integral equations

$$H_1\left\{A(\xi);r\right\}=0 \quad ; \quad 0<r<b \tag{21}$$

$$H_1\left\{\xi^{-1}A(\xi);r\right\}=f(r) \quad ; \quad b\le r\le a \tag{22}$$

$$H_1\left\{A(\xi);r\right\}=0 \quad ; \quad a<r<\infty \tag{23}$$

where $f(r)=\Omega r$. The solution of triple integral equations of the type (21) to (23) can be approached using a variety of computational and approximate analytical schemes. Extensive accounts are given by Tranter (1960), Williams (1963), Cooke (1963), Collins (1963), Sneddon (1966) and Jain and Kanwal (1972). Solution of triple integral equations resulting from annular inclusion and crack problems are also given by Selvadurai and Singh (1984 a, b ; 1985), Selvadurai (1994, 1996) and Singh et al. (1995). We follow the procedure outlined by Selvadurai and Singh (1984a) and assume that the function $A(\xi)$ admits a representation

$$H_1\left\{A(\xi);r\right\}=g(r) \quad ; \quad b<r<a \tag{24}$$

From the Hankel inversion theorem

$$A(\xi)=\int_a^b r\,g(r)\,J_1(\xi r)\,dr \tag{25}$$

Considering (22) and (25), we have

$$\int_a^b u\, g(u)\, K(u,r)\, du = f(r) \quad ; \quad b \le r \le a \tag{26}$$

where

$$K(u,r) = u \int_0^\infty J_1(\xi r)\, J_1(\xi u)\, d\xi \tag{27}$$

We now introduce functions $g_1(u)$ and $g_2(u)$ such that

$$g_1(u) + g_2(u) = \begin{cases} 0 \quad ; & 0 \le r < b \\ g(r); & b \le r \le a \\ 0 \quad ; & a < r < \infty \end{cases} \tag{28}$$

and assume that $f(r)$ admits expansions of the form

$$f_1(r) = \sum_{n=-\infty}^{\infty} a_n r^n \quad ; \quad 0 < r < a \tag{29}$$

$$f_2(r) = \sum_{n=-\infty}^{-1} \tilde{a}_n r^n \quad ; \quad b < r < \infty \tag{30}$$

Using the results (27) to (30), the integral equation (26) can be expressed in the form of two integral equations

$$\int_0^\infty u\, g_1(u)\, K(u,r)\, du = f_1(r) \quad ; \quad 0 < r < a \tag{31}$$

$$\int_0^\infty u\, g_2(u)\, K(u,r)\, du = f_2(r) \quad ; \quad b < r < \infty \tag{32}$$

The solution of the integral equations (31) and (32) can be approached in a variety of ways. Here we adopt the basic approach proposed by Williams (1963) (see also Selvadurai and Singh (1984a)) where these equations can be reduced to the forms

$$4r^{-n} \int_a^r \frac{s^{2n}}{\sqrt{r^2 - s^2}} ds \int_s^\infty \frac{t^{1-n} g_1(t)}{\sqrt{t^2 - s^2}} dt = f_1(r) \quad ; \quad 0 < r < a \tag{33}$$

$$4r^{n} \int_r^\infty \frac{s^{-2n}}{\sqrt{s^2 - r^2}} ds \int_0^s \frac{t^{1+n} g_2(t)}{\sqrt{s^2 - t^2}} dt = f_2(r) \quad ; \quad b < r < \infty \tag{34}$$

These integrals can be inverted and expressed as two coupled Fredholm integral equations of the second kind for auxiliary functions $T_1(r)$ and $T_2(r)$, in the forms

$$T_1(r) = l_1(r) + \frac{n!}{r^n \sqrt{\pi}\, \Gamma(n + (3/2))} \int_0^b \frac{u^{n+1} T_2(u)\ {}_2F_1[(1/2), n; n+3/2); (u^2/r^2)] du}{(r^2 - u^2)}; \quad a < r < \infty \tag{35}$$

$$T_2(r) = l_2(r) + \frac{r^{n+1} n!}{\sqrt{\pi}\Gamma(n+(3/2))} \int_a^\infty \frac{u^{-n} T_1(u)\ {}_2F_1[(1/2), n; n+3/2); (r^2/u^2)] du}{(u^2 - r^2)} \quad ; \qquad 0 < r < b \tag{36}$$

where ${}_2F_1$ is the hypergeometric function and $l_1(r)$ and $l_2(r)$ are given by

$$l_1(r) = -\frac{2}{\pi r^n} \int_0^r \frac{t^{2n} dt}{\sqrt{r^2 - t^2}} \frac{d}{dt} \int_t^a \frac{u^{1-n} B_1(u) du}{\sqrt{u^2 - t^2}} \tag{37}$$

$$l_2(r) = \frac{2r^n}{\pi} \int_r^\infty \frac{t^{-2n} dt}{\sqrt{t^2 - r^2}} \frac{d}{dt} \int_b^t \frac{u^{1+n} B_2(u) du}{\sqrt{t^2 - u^2}} \tag{38}$$

and the functions $B_1(r)$ and $B_2(r)$ are given by

$$B_1(r) = \frac{1}{2\pi r^n} \frac{d}{dr} \int_0^r \frac{u^{n+1} f_1(u) du}{\sqrt{r^2 - u^2}} \quad ; \quad 0 < r < a \tag{39}$$

$$B_2(r) = -\frac{r^n}{2\pi} \frac{d}{dr} \int_r^\infty \frac{u^{1-n} f_2(u) du}{\sqrt{u^2 - r^2}} \quad ; \quad b < r < \infty \tag{40}$$

5. Torsional Stiffness of the Annular Punch

The coupled integral equations (35) and (36) can be solved using a variety of numerical techniques (see. e.g. Delves and Mohamed, 1985; Atkinson, 1997). An alternative is to employ an iterative technique that results in an analytical solution to the coupled equations, albeit in an approximate form. The procedure becomes feasible if the form of the solution can be developed in a series involving a small non-dimensional parameter. An inspection of the governing integral equations reveal that a suitable non-dimensional parameter can be set as the radii ratio of the annulus $b/a\ (=\varepsilon)$. For example for the torsional indentation problem, $n=1$; $f(r) = \Omega r$; $f_1(r) = \Omega$ and $f_2(r) = 0$. The integral equations (35) and (36) can be solved, using an iterative procedure to obtain expressions for $T_1(r)$ and $T_2(r)$. These can be used to generate the stresses and displacements. In this paper we are primarily interested in developing an expression for the torsional stiffness for the bonded annular disc. The torque T necessary to induce the rotation Ω is given by the result

$$T = 2\pi \int_b^a \left\{ r\, \overline{\tau}^{23} \right\} r^2 dr \tag{41}$$

which can be evaluated in the form

$$T = \frac{16\Omega a^3 (\Phi + \mu^2 \Psi)}{3\mu^3} \left(1 - \frac{16\varepsilon^5}{15\pi^2} - \frac{64\varepsilon^7}{105\pi^2} + 0(\varepsilon^9) \right) \sqrt{\frac{\Psi + \mu^4 \Phi}{\Phi + \mu^2 \Psi}} \tag{42}$$

where ε is the radii ratio. In the special case when the strain energy function for the elastic material has a Mooney-Rivlin form: $W = C_1(I_1 - 3) + C_2(I_2 - 3)$, the result (42) reduces to

$$T = \frac{32\Omega a^3 (C_1 + \mu^2 C_2)}{3\mu^3} \left(1 - \frac{16\varepsilon^5}{15\pi^2} - \frac{64\varepsilon^7}{105\pi^2} + 0(\varepsilon^9) \right) \sqrt{\frac{\Gamma + \mu^4}{1 + \mu^2 \Gamma}} \tag{43}$$

where $\Gamma = C_2 / C_1$. Also in the absence of the initial finite deformation, (i.e. $\mu \to 1$) and when the indenter is a solid circular disc (i.e. $\varepsilon \to 0$), (43) reduces to the classical Reissner and Sagoci (1944) and Sneddon (1944) result

$$T = \frac{16 G \Omega a^3}{3} \tag{44}$$

where $G(= 2(C_1 + C_2))$ is the linear elastic shear modulus. Since the result for the torsional stiffness has been presented in explicit form, the result can be evaluated for ranges of values of μ and ε of interest. As has been observed by Green et al. (1952), Woo and Shield (1961), Beatty and Usmani (1975) and Selvadurai (1977), instability at the surface of a radially compressed incompressible elastic halfspace can occur when $\mu \approx 2/3$. Accordingly, the minimum radial compression should correspond to this limit. Also, the result $\Gamma = 0$, corresponds to the neo-Hookean elastic material.

6. Concluding Remarks

The theory of small deformations superposed on large developed by Green et al. (1952) provides a valuable approach for examining initial stress effects within the context of hyper elastic materials. The problem examined in this paper combines the approaches available in the literature for solving three-part mixed boundary value problems in classical elasticity with the theory of small deformations superposed on large to examine the torsional indentation problem for the annular indenter. An analytical result has some practical value in that the torsional stiffness can be conveniently evaluated using the approximate result where the influence of the annular configuration of the indenter is accounted for through a series in terms of a small non-dimensional parameter that represents the radii ratio.

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